The variation of R_{opt} with J is shown in Fig. 6 with $A = 0.01$ and n as a parameter. For a particular value of J (i.e. heat load), there is an optimum value of R which should be used. In other words, using

$$\Delta P = \frac{f\rho U^2 L}{r} \tag{19}$$

in equation (15)

$$U_{opt} = \left[\frac{Q}{\pi r f \rho L R_{opt}}\right]^{1/3}. \tag{20}$$

Thus there is an optimum fluid velocity which corresponds to the minimum loss of available power and should be recommended in the design of the heat exchanger.

4. CONCLUSIONS

This analysis shows that in any heat transfer application with the constant wall temperature boundary condition, the initial temperature difference between the fluid and the wall is an important design criterion and should be set at the optimum value. There is an optimum ratio of the heat transfer to pumping power which should be used. Simply maximizing this ratio is not often a good solution, since in that case the entropy generated may be far from the minimum possible and a large amount of the available energy may thus be irretrievably lost. An optimum fluid velocity corresponding to the minimum irreversibility is recommended for the design of such a heat exchanger.

REFERENCES

1. A. Bejan, General criterion for rating heat exchanger performance, *Int. J. Heat Mass Transfer* **21**, 665 (1978).
2. A. Bejan, A study of entropy generation in fundamental convective heat transfer, *J. Heat Transfer* **101**, 718 (1979).
3. A. Bejan, The concept of irreversibility in heat exchanger design: counterflow heat exchanger for gas-to-gas applications, *J. Heat Transfer* **99**, 374 (1977).
4. A. Bejan and A. L. Smith, Thermodynamic optimization of mechanical supports for cryogenic apparatus, *Cryogenics* **14**, 158 (1974).
5. A. Bejan and A. L. Smith, Heat exchangers for vapour cooled conducting supports of cryostats, *Adv. Cryogenic Engng* **21**, 247 (1975).
6. S. Sarangi and K. Chowdhury, On the generation of entropy in counterflow heat exchangers, *Cryogenics* **22**, 63 (1982).
7. P. J. Golem and T. A. Brzustowski, Second law analysis of thermal processes, *Trans. CSME* **4**, 219 (1977).
8. J. E. Parrot, Theoretical upper limit to the conversion efficiency of solar energy, *Sol. Energy* **21**, 227 (1978).
9. J. F. Kreider, Second law analysis of solar thermal processes, *Energy Res.* **3**, 325 (1979).
10. A. L. London, Economics and the second law, *Int. J. Heat Mass Transfer* **25**, 743 (1982).
11. E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*, p. 373. McGraw-Hill Kogakusha, Tokyo (1972).

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Correlations for laminar mixed convection on vertical, inclined and horizontal flat plates with uniform surface heat flux

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INTRODUCTION

MIXED convection accounts for the buoyancy effects on forced flows or the forced flow effects on buoyant flows. Published results on mixed convection flows do not cover the entire mixed convection regime and, in addition, the uniform wall temperature case (UWT) has received significantly more attention than the uniform wall heat flux case (UHF). A relatively comprehensive summary on mixed convection in external flows has been given recently by Chen *et al.* [1].

To summarize the analytical studies for mixed convection adjacent to flat plates under the UHF heating condition, it is noted that the local Nusselt number results have been presented for vertical plates covering $0 \leqslant Gr_x^*/Re_x^{5/2} \leqslant 2.8$ for $0.1 \leqslant Pr \leqslant 100$ [2], inclined plates covering $-0.25 \leqslant Gr_x^* \cos\gamma/Re_x^{5/2} \leqslant 5$ for $Pr = 0.7$ and 7 [3] and $-1 \leqslant Gr_x^*/Re_x^{5/2} \leqslant 2$ for $Pr = 0.7$ and 7 [4], and horizontal plates covering $0 \leqslant Gr_x^*/Re_x^3 \leqslant 1$ for $Pr = 0.7$ [5]. Thus, it is clear that the heat transfer results that have been presented for the UHF case are rather limited in scope with regard to the ranges of buoyancy parameter Gr_x^*/Re_x^m and Prandtl numbers.

In the present paper, comprehensive results for the local and average Nusselt numbers are presented for the entire mixed convection regime, ranging from pure forced convection to pure free convection (i.e. for $0 \leqslant Gr_x^*/Re_x^m \leqslant \infty$), for a wide range of Prandtl numbers, $0.1 \leqslant Pr \leqslant 100$. The flow configurations covered include vertical, inclined and horizontal flat plates with uniform surface heat flux. Both buoyancy assisting and opposing flows are treated. The upper and lower bounds (a, b) of the significant mixed convection regime, $a \leqslant Gr_x/Re_x^m \leqslant b$, are established. In addition, simple correlation equations for the local and average Nusselt numbers are formulated for all the flow configurations. Such a comprehensive treatment of mixed convection flows on flat plates has not been carried out for the UHF case.

CORRELATIONS

The formulation and the treatment of laminar mixed convection flow adjacent to a semi-infinite flat plate with uniform heat flux, q_w, imposed on its surface have been presented for vertical, inclined and horizontal geometries [1]. That formulation was used to generate new numerical results for these flow configurations which cover the entire mixed convection regime for the buoyancy assisting and the buoyancy opposing flow conditions as shown in Figs. 1 and 2. These results were used to validate the accuracy of proposed simple correlations for the local and average mixed convection Nusselt numbers.

NOMENCLATURE

g gravitational acceleration
Gr_x^* modified local Grashof number, $g\beta q_w x^4/kv^2$
Gr_L^* modified Grashof number based on L, $g\beta q_w L^4/kv^2$
h local heat transfer coefficient, $q_w/(T_w - T_\infty)$
$\bar{h}$ average heat transfer coefficient, $\frac{1}{L}\int_0^L h\,dx$
k thermal conductivity
L length of plate
n constant exponent
Nu_F, Nu_N, Nu_x local Nusselt numbers for pure forced, pure free, and mixed convection, hx/k
$\overline{Nu}_F$, $\overline{Nu}_N$, $\overline{Nu}$ average Nusselt numbers for pure forced, pure free, and mixed convection, $\bar{h}L/k$
Pr Prandtl number
q local surface heat flux
Re_x local Reynolds number, $u_\infty x/v$
Re_L Reynolds number based on L, $u_\infty L/v$
T fluid temperature
u, v streamwise and normal velocity components
x, y streamwise and normal coordinates.

Greek symbols
α thermal diffusivity
β volumetric coefficient of thermal expansion
γ angle of inclination from the vertical
v kinematic viscosity.

Subscripts
w condition at wall
∞ condition at free stream.

Similar to the previous treatment of the uniform wall temperature case [6] for this geometry, a correlation for the local mixed convection Nusselt number for the UHF, boundary condition, Nu_x, is proposed in terms of the local Nusselt number of the pure forced convection, Nu_F, and the local Nusselt number of the pure free convection, Nu_N, for the same geometry and boundary condition as [7]

$$Nu_x^n = Nu_F^n \pm Nu_N^n. \tag{1}$$

In the above equation, n is a constant exponent and the plus and minus signs pertain respectively to buoyancy assisting and buoyancy opposing flows. Equation (1) can be expressed as

$$Y^n = 1 \pm X^n \tag{2}$$

where

$$Y = Nu_x/Nu_F, \quad X = Nu_N/Nu_F. \tag{3}$$

Equations (2) and (3) can also be applied to the average Nusselt number correlation if the Nu_x, Nu_F and Nu_N expressions in the equations are replaced with the corresponding average Nusselt number expressions $\overline{Nu}$, $\overline{Nu}_F$ and $\overline{Nu}_N$, respectively. Correlations equivalent to equation (2) for vertical, horizontal and inclined plates are presented below.

Vertical flat plates

The local Nusselt number expression for the pure forced convection in laminar boundary-layer flow along a vertical flat plate under the UHF boundary condition is given by [8]

$$Nu_F = G_1(Pr)Re_x^{1/2},$$
$$G_1(Pr) = 0.464Pr^{1/3}[1+(0.0207/Pr)^{2/3}]^{-1/4}. \tag{4}$$

The corresponding expression for the pure free convection is given by [9]

$$Nu_N = G_2(Pr)Gr_x^{*1/5},$$
$$G_2(Pr) = Pr^{2/5}(4+9Pr^{1/2}+10Pr)^{-1/5}. \tag{5}$$

The local Nusselt number for mixed convection flow can then be expressed according to equation (2) as

$$Nu_x Re_x^{-1/2}/G_1(Pr) = \{1 \pm [G_2(Pr) \times (Gr_x^*/Re_x^{5/2})^{1/5}/G_1(Pr)]^n\}^{1/n}. \tag{6}$$

Similarly, the mixed convection average Nusselt numbers can be correlated as

$$\overline{Nu}Re_L^{-1/2}/2G_1(Pr) = \{1 \pm [5G_2(Pr) \times (Gr_L^*/Re_L^{5/2})^{1/5}/8G_1(Pr)]^n\}^{1/n}. \tag{7}$$

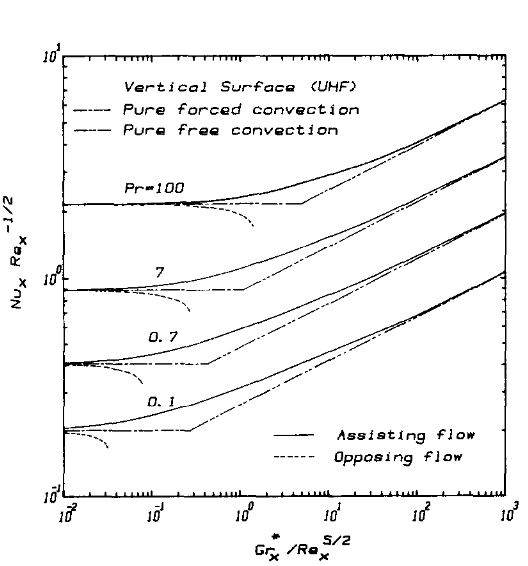

FIG. 1. Local Nusselt number results for flow along a vertical flat plate with uniform surface heat flux.

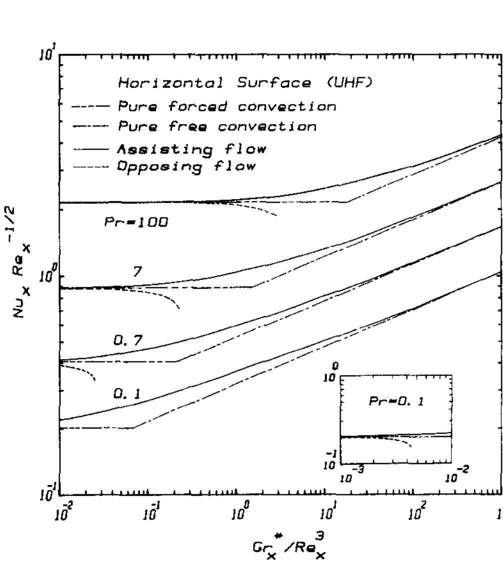

FIG. 2. Local Nusselt number results for flow over a horizontal flat plate with uniform surface heat flux.

It is seen that equations (6) and (7) have the form $Y = (1 \pm X^n)^{1/n}$.

Horizontal flat plates

The correlation equation for the local Nusselt number for the pure forced convection limit, Nu_F, for this flow configuration is given by equation (4). The expression for the pure natural convection limit, Nu_N, for an upward facing, UHF-heated flat plate was developed from the present calculations and has the following form

$$Nu_N = G_3(Pr)Gr_x^{*1/6},$$
$$G_3(Pr) = (Pr/6)^{1/6}Pr^{1/2}(0.12+1.2Pr^{1/2})^{-1}. \quad (8)$$

The local Nusselt number in mixed convection flow for this geometry according to equation (2) then has the expression

$$Nu_x Re_x^{-1/2}/G_1(Pr) = \{1 \pm [G_3(Pr) \times (Gr_x^*/Re_x^3)^{1/6}/G_1(Pr)]^n\}^{1/n}. \quad (9)$$

The corresponding expression for the average Nusselt number assumes the form

$$\overline{Nu}Re_L^{-1/2}/2G_1(Pr) = \{1 \pm [3G_3(Pr) \times (Gr_L^*/Re_L^3)^{1/6}/4G_1(Pr)]^n\}^{1/n}. \quad (10)$$

Inclined flat plates

As was discussed in refs. [3, 6], equations (6) and (7) for the vertical plates can be used with good accuracy for the inclined plates in the angle range of $0° \leqslant \gamma \leqslant 75°$ by simply replacing Gr_x^* and Gr_L^* in these equations with $Gr_x^* \cos\gamma$ and $Gr_L^* \cos\gamma$, respectively, when the Reynolds numbers are larger than 10^3.

For larger inclination angles $75° \leqslant \gamma \leqslant 90°$ different correlations are recommended for the mixed convection Nusselt numbers. The local Nusselt number for pure free convection along a UHF plate in the inclination angle range of $75° \leqslant \gamma \leqslant 90°$ has been correlated by Chen *et al.* [10] as

$$Nu_N = G_4(Pr)Gr_x^{*1/6+D(\gamma)}, \quad 10^4 \leqslant Gr_x^* Pr \leqslant 10^{10} \quad (11)$$

where

$$G_4(Pr) = Pr^{1/2}(0.12+1.2Pr^{1/2})^{-1}(Pr/6)^{1/6+D(\gamma)} \quad (12)$$

and

$$D(\gamma) = 0.038(\cos\gamma)^{1/2}. \quad (13)$$

The correlation for the local mixed convection Nusselt number in the inclination angle range of $75° \leqslant \gamma \leqslant 90°$ is then given by

$$Nu_x Re_x^{-1/2}/G_1(Pr) = \{1 \pm [G_4(Pr) \times (Gr_x^*/Re_x^3)^{1/6} Gr_x^{*D(\gamma)}/G_1(Pr)]^n\}^{1/n}. \quad (14)$$

Similarly, the average mixed convection Nusselt number for $75° \leqslant \gamma \leqslant 90°$ can be correlated as

$$\overline{Nu}Re_L^{-1/2}/2G_1(Pr) = \left\{1 \pm \left[\frac{G_4(Pr)(Gr_L^*/Re_L^3)^{1/6}Gr_L^{*D(\gamma)}}{8[1/6+D(\gamma)]G_1(Pr)}\right]^n\right\}^{1/n}. \quad (15)$$

RESULTS AND DISCUSSION

The predicted local mixed convection Nusselt numbers, $Nu_x Re_x^{-1/2}$, are presented in Fig. 1 as a function of the buoyancy parameter, $Gr_x^*/Re_x^{5/2}$, for the vertical plate geometry. Similar results are presented in Fig. 2 for the horizontal plate geometry in terms of the corresponding buoyancy parameter, Gr_x^*/Re_x^3. It has been verified from calculations for the inclined plate that Fig. 1 can also be used with good accuracy for inclined plates in the angle range of $0° \leqslant \gamma \leqslant 75°$ if the abscissa in that figure is replaced by $Gr_x^* \cos\gamma/Re_x^{5/2}$. It is clear from these two figures that mixed convection Nusselt numbers are larger than either the pure forced or pure free convection values for buoyancy assisting flows and that they are smaller for buoyancy opposing flows. In the latter case, buoyancy forces cause a breakdown in the boundary-layer flow due to flow separation at small values of the buoyancy parameter. In addition, fluids with lower Prandtl numbers are seen to exhibit a higher sensitivity to increasing buoyancy forces in comparison to fluids with higher Prandtl numbers. Thus, the mixed convection regime changes as the Prandtl number changes. This mixed convection regime can be quantified by specifying the upper and lower bounds (a and b), $a \leqslant Gr_x^*/Re_x^m \leqslant b$, based on a 5% departure in the local Nusselt number from the pure forced and the pure free convection asymptotes. The upper and lower bounds, a and b, are listed in Table 1 along with the maximum deviation from the pure convection asymptotes.

The validity of the proposed simple correlations, equation (6) for vertical plates and equation (9) for horizontal plates, with an exponent of $n = 3$ was tested by comparing the local Nusselt numbers resulting from these correlations with the numerically predicted values that are shown in Figs. 1 and 2. The maximum deviation between the correlated and the predicted values for both flow configurations is less than 5% for assisting flow and about 10% for opposing flow for the range of Prandtl numbers $0.1 \leqslant Pr \leqslant 100$. The correlations for the vertical plates, equations (6) and (7), can be used with good accuracy to calculate the Nusselt numbers for an inclined plate with $0° \leqslant \gamma < 90°$ by replacing the Grashof numbers, Gr_x^* and Gr_L^* in these correlations with effective Grashof numbers $Gr_x^* \cos\gamma$ and $Gr_L^* \cos\gamma$, respectively.

The results for all flow configurations can be presented in one figure if the coordinates Y and X can be selected for the appropriate geometry as demonstrated in Fig. 3 for the case of $Pr = 0.7$ for all angles of inclination $0° \leqslant \gamma \leqslant 90°$. It is clear from this figure that the proposed simple correlations for the mixed convection regime compare very well with the numerically predicted results. It should be noted that in Fig. 3 the variables $A(Pr)$, $B(Pr)$ and ξ^m stand for $G_1(Pr)$, $G_2(Pr)$ and $(Gr_x^*/Re_x^{5/2})^{1/5}$ for a vertical plate, for $G_1(Pr)$, $G_2(Pr)$ and $(Gr_x^* \cos\gamma/Re_x^{5/2})^{1/5}$ or $G_1(Pr)$, $G_4(Pr)$ and $(Gr_x^*/Re_x^3)^{1/6}Gr_x^{*D(\gamma)}$ for an inclined plate, and for $G_1(Pr)$, $G_3(Pr)$ and $(Gr_x^*/Re_x^3)^{1/6}$ for a horizontal plate.

Simple correlations for the average mixed convection Nusselt numbers for the vertical, inclined and horizontal plate geometries are given in equations (7), (10) and (15). A good agreement between the predicted values and the correlated

Table 1. Lower/upper bounds of significant buoyancy/forced-flow effects and maximum percentage increase in the local Nusselt number

	Vertical/inclined plates $a \leqslant Gr_x^* \cos\gamma/Re_x^{5/2} \leqslant b$			Horizontal plates $a \leqslant Gr_x^*/Re_x^3 \leqslant b$		
Prandtl number, Pr	a	b	Max. increase in Nu_x (%)	a	b	Max. increase in Nu_x (%)
0.1	0.02	15	33	0.005	18	29
0.7	0.04	20	27	0.025	20	22
7	0.15	22	24	0.15	40	22
100	0.70	70	24	2.5	400	22

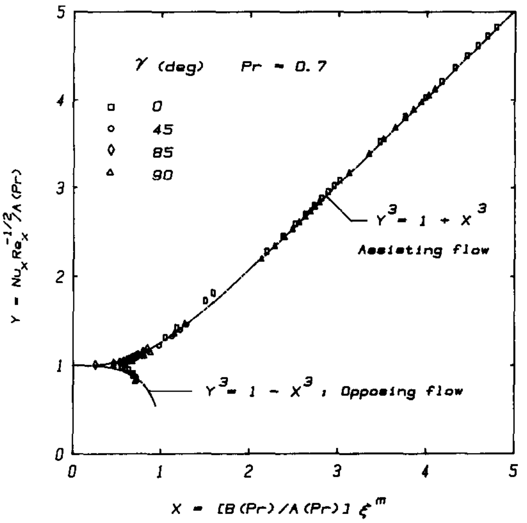

FIG. 3. A comparison between predicted and correlated local Nusselt numbers for vertical, inclined and horizontal plates.

results was also found to exist for the average Nusselt number with an exponent value of $n = 3$, as in the local Nusselt number.

CONCLUSION

The local and average Nusselt numbers for laminar mixed convection flow adjacent to vertical, inclined and horizontal flat plates with uniform surface heat flux are presented for the entire mixed convection regime and for a wide range of Prandtl numbers. Simple correlation equations for Nusselt numbers are presented, which show an excellent agreement with the numerically predicted mixed convection values for both buoyancy assisting and opposing flows.

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REFERENCES

1. T. S. Chen, B. F. Armaly and W. Aung, Mixed convection in laminar boundary-layer flow. In *Natural Convection: Fundamentals and Applications* (Edited by S. Kakac *et al.*), pp. 699–725. Hemisphere, Washington, DC (1985).
2. G. Wilks, Combined forced and free convection flow on vertical surfaces, *Int. J. Heat Mass Transfer* **16**, 1958–1964 (1973).
3. A. Mucoglu and T. S Chen, Mixed convection on inclined surfaces, *J. Heat Transfer* **101**, 422–426 (1979).
4. A. Moutsoglou, K. L. Tzuoo and T. S Chen, Mixed convection in boundary layer flows over inclined surfaces. Presented at the AIAA 15th Thermophysics Conference, Snowmass, CO, Paper No. AIAA-80-1525 (July 1980).
5. T. S. Chen, E. M. Sparrow and A. Mucoglu, Mixed convection in boundary layer flow on a horizontal plate, *J. Heat Transfer* **99**, 66–71 (1977).
6. T. S. Chen, B. F. Armaly and N. Ramachandran, Correlations for laminar mixed convection flows on vertical, inclined, and horizontal flat plates, *J. Heat Transfer* (in press).
7. S. W. Churchill, A comprehensive correlating equation for laminar, assisting, forced and free convection, *A.I.Ch.E. Jl* **23**, 10–16 (1977).
8. S. W. Churchill and H. Ozoe, Correlations for laminar forced convection with uniform surface heating in flow over a plate and in developing and fully developed flow in a tube, *J. Heat Transfer* **95**, 78–84 (1973).
9. T. Fujii and M. Fujii, The dependence of local Nusselt number on Prandtl number in the case of free convection along a vertical surface with uniform heat flux, *Int. J. Heat Mass Transfer* **19**, 121–122 (1976).
10. T. S. Chen, H. C. Tien and B. F. Armaly, Natural convection on horizontal, inclined, and vertical plates with variable surface temperature or heat flux, *Int. J. Heat Mass Transfer* **29**, 1465–1478 (1986).

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Large amplitude modulation of heat transfer from a circular cylinder

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1. INTRODUCTION

THE UNSTEADY heat transfer from a heated wire is encountered in various applications including hot-wire or film anemometry and electronic cooling. The time lag between the heat transfer and the relative velocity of the fluid to the wire is now well recognized. Using the Oseen approximation, Davies [1] analyzed the heat transfer from a constant-temperature circular cylinder in a cross-flow which has a small, sinusoidally fluctuating velocity superimposed on the mean velocity. The Reynolds number corresponding to the mean flow considered by him was smaller than one. Davies found that there is always a phase lag between the fluctuating velocity and the fluctuating heat transfer unless when the Reynolds number approaches zero. Apelt and Ledwich [2] studied the same problem for flows of Reynolds numbers in the range 1–40. They found numerically that the phase lag becomes more pronounced as the frequency of cylinder oscillation increases. Tseng and Lin [3] showed, by use of an asymptotic solution, that the phase lag persists in flows of Reynolds number of a few hundred. They also found the existence of an optimal frequency for the maximum heat transfer enhancement in a cross-flow of a given mean flow Reynolds number and a given small amplitude of cylinder fluctuation. Their asymptotic solution was later applied to construct the theory of a heat sensing velocimeter [4]. In the present investigation, we are concerned with the unsteady heat transfer from a heated cylinder oscillating with a large